

# CP Algebra 2: HW Key [Gaussian Elimination]

① 
$$\begin{cases} 4x - 5y = 7 \\ 2x + 7y = 13 \end{cases}$$

$R_1 \leftrightarrow R_2 \rightarrow \begin{cases} 2x + 7y = 13 \\ 4x - 5y = 7 \end{cases}$

$\frac{1}{2}R_1 \rightarrow R_1 \rightarrow \begin{cases} x + \frac{7}{2}y = \frac{13}{2} \\ 4x - 5y = 7 \end{cases}$

$$\begin{array}{r} -4x - 14y = -26 \\ 4x - 5y = 7 \\ \hline -19y = -19 \end{array}$$

$-4R_1 + R_2 \rightarrow R_2 \rightarrow \begin{cases} x + \frac{7}{2}y = \frac{13}{2} \\ -19y = -19 \end{cases}$

$-19y = -19$

$y = 1$

$x + \frac{7}{2}y = \frac{13}{2}$

$x + \frac{7}{2}(1) = \frac{13}{2}$

$x + \frac{7}{2} = \frac{13}{2}$

$x = 3$

$$\begin{cases} x = 3 \\ y = 1 \end{cases}$$

② 
$$\begin{cases} 5x - 7y = 12 \\ 3x + 2y = 8 \end{cases}$$

$\frac{1}{5}R_1 \rightarrow R_1 \rightarrow \begin{cases} x - \frac{7}{5}y = \frac{12}{5} \\ 3x + 2y = 8 \end{cases}$

$$\begin{array}{r} -3x + \frac{21}{5}y = -\frac{36}{5} \\ 3x + 2y = 8 \\ \hline \frac{31}{5}y = \frac{4}{5} \end{array}$$

$-3R_1 + R_2 \rightarrow R_2 \rightarrow \begin{cases} x - \frac{7}{5}y = \frac{12}{5} \\ \frac{31}{5}y = \frac{4}{5} \end{cases}$

$\frac{31}{5}y = \frac{4}{5}$

$y = \frac{4}{31}$

$x - \frac{7}{5}y = \frac{12}{5}$

$x - \frac{7}{5}\left(\frac{4}{31}\right) = \frac{12}{5}$

$x - \frac{28}{155} = \frac{12}{5}$

$x = \frac{80}{31}$

$$\begin{cases} x = \frac{80}{31} \\ y = \frac{4}{31} \end{cases}$$

(3)

$$\begin{cases} x-3y+z=44 \longrightarrow -x+3y-z=-44 \\ x-y-z=26 \longrightarrow \underline{x-y-z=26} \\ 5x+2y+19z=58 \end{cases} \quad \begin{array}{l} \\ \\ \hline 2y-2z=-18 \end{array}$$

$$\xrightarrow{-R_1+R_2 \rightarrow R_2} \begin{cases} x-3y+z=44 \longrightarrow -5x+15y-5z=-220 \\ 2y-2z=-18 \longrightarrow \underline{5x+2y+19z=58} \\ 5x+2y+19z=58 \end{cases} \quad \begin{array}{l} \\ \\ \hline 17y+14z=-162 \end{array}$$

$$\xrightarrow{-5R_1+R_3 \rightarrow R_3} \begin{cases} x-3y+z=44 \\ 2y-2z=-18 \\ 17y+14z=-162 \end{cases}$$

$$\xrightarrow{\frac{1}{2}R_2 \rightarrow R_2} \begin{cases} x-3y+z=44 \\ y-z=-9 \longrightarrow -17y+17z=153 \\ 17y+14z=-162 \longrightarrow \underline{17y+14z=-162} \\ \hline 31z=-9 \end{cases}$$

$$\xrightarrow{-17R_2+R_3 \rightarrow R_3} \begin{cases} x-3y+z=44 \\ y-z=-9 \\ 31z=-9 \end{cases} \longrightarrow \begin{cases} y-z=-9 \\ y-(-\frac{9}{31})=-9 \\ y+\frac{9}{31}=-9 \\ \hline y=-\frac{288}{31} \end{cases}$$

$$\boxed{z = -\frac{9}{31}} \longrightarrow \boxed{y = -\frac{288}{31}}$$

$$\begin{aligned} x-3y+z &= 44 \\ x-3\left(-\frac{288}{31}\right) + \left(-\frac{9}{31}\right) &= 44 \\ x + \frac{855}{31} &= 44 \\ \boxed{x = \frac{509}{31}} \end{aligned}$$

$$\boxed{\begin{aligned} x &= \frac{509}{31} \\ y &= -\frac{288}{31} \\ z &= -\frac{9}{31} \end{aligned}}$$

One  
option:  
Must  
follow  
method  
though!

④

$$\begin{cases} 7x - 5y + 3z = 182 \\ x + 4y - 2z = -112 \\ -5x + y - 11z = -238 \end{cases}$$

One option:  
Must follow  
method  
though!

$R_1 \leftrightarrow R_2$   
→

$$\begin{cases} x + 4y - 2z = -112 \rightarrow -7x - 28y + 14z = 784 \\ 7x - 5y + 3z = 182 \rightarrow 7x - 5y + 3z = 182 \\ -5x + y - 11z = -238 \end{cases}$$


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$$-33y + 17z = 966$$

$-7R_1 + R_2 \rightarrow R_2$   
→

$$\begin{cases} x + 4y - 2z = -112 \rightarrow 5x + 20y - 10z = -560 \\ -33y + 17z = 966 \\ -5x + y - 11z = -238 \end{cases}$$


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$$21y - 21z = -798$$

$+5R_1 + R_3 \rightarrow R_3$   
→

$$\begin{cases} x + 4y - 2z = -112 \\ -33y + 17z = 966 \\ 21y - 21z = -798 \end{cases}$$

$R_2 \leftrightarrow R_3$   
→

$$\begin{cases} x + 4y - 2z = -112 \\ 21y - 21z = -798 \\ -33y + 17z = 966 \end{cases}$$

$\frac{1}{21}R_2 \rightarrow R_2$   
→

$$\begin{cases} x + 4y - 2z = -112 \\ y - z = -38 \rightarrow 33y - 33z = -1254 \\ -33y + 17z = 966 \rightarrow -33y + 17z = 966 \end{cases}$$


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$$-16z = -288$$

$33R_2 + R_3 \rightarrow R_3$   
→

$$\begin{cases} x + 4y - 2z = -112 \\ y - z = -38 \\ -16z = -288 \end{cases}$$

$$-16z = -288$$

$$z = 18$$

$$y - z = -38$$

$$y - 18 = -38$$

$$y = -20$$

$$x + 4y - 2z = -112$$

$$x + 4(-20) - 2(18) = -112$$

$$x - 80 - 36 = -112$$

$$x - 116 = -112$$

$$x = 4$$

5

$$\begin{cases} 7x - 12y + 5z = 10 \\ -2x + 8y - z = -30 \\ 2x - 3y + 4z = 50 \end{cases}$$

One option:  
Must follow  
method though!

$$\xrightarrow{R_1 \leftrightarrow R_3} \begin{cases} 2x - 3y + 4z = 50 \\ -2x + 8y - z = -30 \\ 7x - 12y + 5z = 10 \end{cases}$$

$$\xrightarrow{\frac{1}{2}R_1 \rightarrow R_1} \begin{cases} x - \frac{3}{2}y + 2z = 25 \\ -2x + 8y - z = -30 \\ 7x - 12y + 5z = 10 \end{cases} \rightarrow \begin{array}{l} 2x - 3y + 4z = 50 \\ -2x + 8y - z = -30 \\ \hline 5y + 3z = 20 \end{array}$$

$$\xrightarrow{2R_1 + R_2 \rightarrow R_2} \begin{cases} x - \frac{3}{2}y + 2z = 25 \\ 5y + 3z = 20 \\ 7x - 12y + 5z = 10 \end{cases} \rightarrow \begin{array}{l} -7x + \frac{21}{2}y - 14z = -175 \\ 7x - 12y + 5z = 10 \\ \hline -\frac{3}{2}y - 9z = -165 \end{array}$$

$$\xrightarrow{-7R_1 + R_3 \rightarrow R_3} \begin{cases} x - \frac{3}{2}y + 2z = 25 \\ 5y + 3z = 20 \\ -\frac{3}{2}y - 9z = -165 \end{cases}$$

$$\xrightarrow{\frac{1}{5}R_2 \rightarrow R_2} \begin{cases} x - \frac{3}{2}y + 2z = 25 \\ y + \frac{3}{5}z = 4 \\ -\frac{3}{2}y - 9z = -165 \end{cases} \rightarrow \begin{array}{l} \frac{3}{2}y + \frac{9}{10}z = 6 \\ -\frac{3}{2}y - 9z = -165 \\ \hline -\frac{81}{10}z = -159 \end{array}$$

$$\xrightarrow{\frac{3}{2}R_2 + R_3 \rightarrow R_3} \begin{cases} x - \frac{3}{2}y + 2z = 25 \\ y + \frac{3}{5}z = 4 \\ -\frac{81}{10}z = -159 \end{cases}$$

$$\begin{array}{l} \swarrow \\ -\frac{81}{10}z = -159 \\ \boxed{z = \frac{530}{27}} \end{array} \rightarrow \begin{array}{l} y + \frac{3}{5}z = 4 \\ y + \frac{3}{5}\left(\frac{530}{27}\right) = 4 \\ y + \frac{106}{9} = 4 \\ \boxed{y = -\frac{70}{9}} \end{array}$$

$$x - \frac{3}{2}y + 2z = 25$$

$$x - \frac{3}{2}\left(-\frac{70}{9}\right) + 2\left(\frac{530}{27}\right) = 25$$

$$x + \frac{1375}{27} = 25$$

$$\boxed{x = -\frac{700}{27}}$$