1.) Find the quotient and remainder when $(x^6+5x^5-7x^4+22x^2-5x-7)$ is divided by (x^2+5x-4). Then, write the original expression in the divisor, quotient, and remainder form. **[6 Points]**

①

$$\begin{array}{r|l} & x^4 \qquad -3x^2+15x-65 \\ \hline x^2+5x-4 & x^6+5x^5-7x^4+0x^3+22x^2-5x-7 \\ & -(x^6+5x^5-4x^4) \\ \hline & \qquad -3x^4+0x^3+22x^2-5x-7 \\ & \quad -(-3x^4-15x^3+12x^2) \\ \hline & \qquad\qquad 15x^3+10x^2-5x-7 \\ & \qquad\quad -(15x^3+75x^2-60x) \\ \hline & \qquad\qquad\qquad -65x^2+55x-7 \\ & \qquad\qquad -(-65x^2-325x+260) \\ \hline & \qquad\qquad\qquad\qquad 380x-267 \end{array}$$

$$\boxed{x^6+5x^5-7x^4+22x^2-5x-7=(x^2+5x-4)\underset{\substack{\uparrow \\ \text{Quotient}}}{(x^4-3x^2+15x-65)}+\underset{\substack{\uparrow \\ \text{Remainder}}}{380x-267}}$$

2.) What is the remainder when $(99x^{57}-5x^{28}+3x^{21}-4x^{12}+x-121)$ is divided by $(x+1)$? **[3 Points]**

② Plug in $x=-1$:

$$99(-1)^{57}-5(-1)^{28}+3(-1)^{21}-4(-1)^{12}+(-1)-121$$

$$=-99-5-3-4-1-121$$

$$\boxed{=-233}$$

---

③ See graphs on the next 2 pages

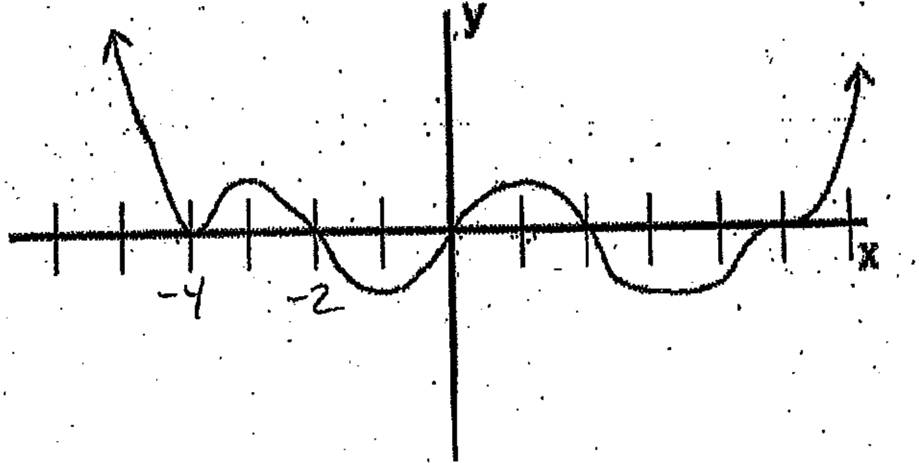
3

Advanced Precalculus: Mr. Hamilton **Name:** Key

Written Assignment #2

Appendix: Figure 1 (Front and Back)

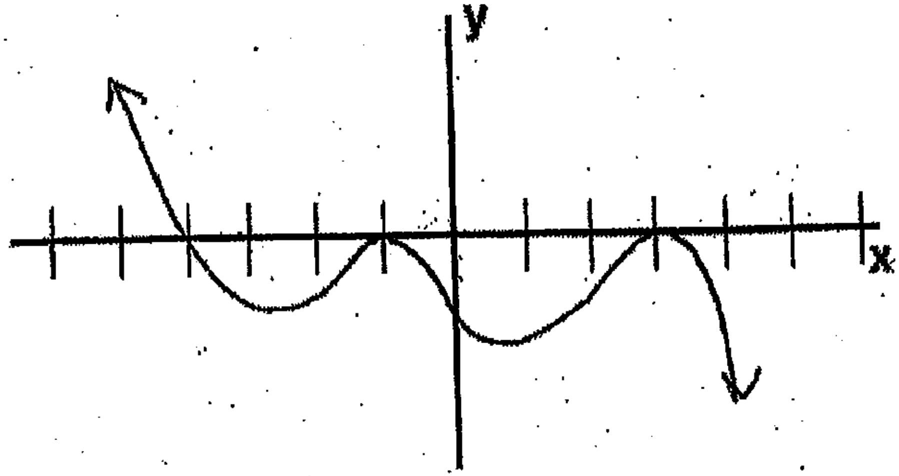
Figure 1.1 Degree 8



Zeros: $x = -4, -2, 0, 2, 5$

Factored: $f(x) = (x+4)^2(x+2)(x)(x-2)(x-5)^3$

Figure 1.2 Degree 5

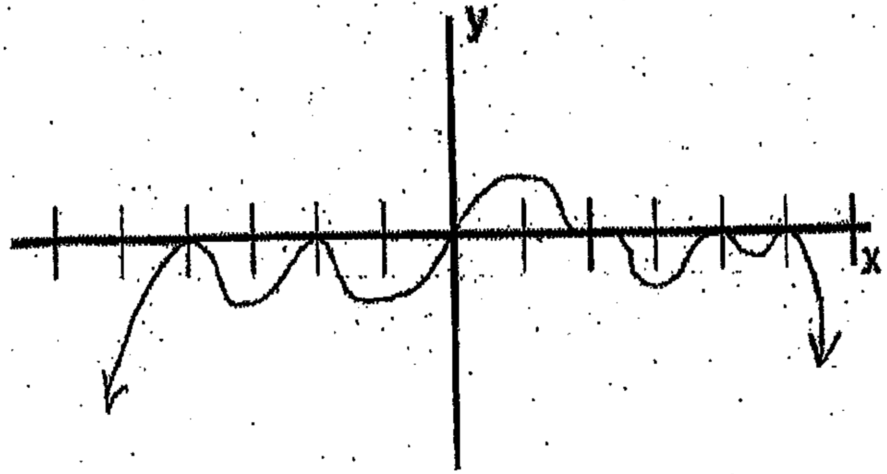


Zeros: $x = -4, -1, 3$

Factored: $f(x) = -(x+4)(x+1)^2(x-3)^2$

③

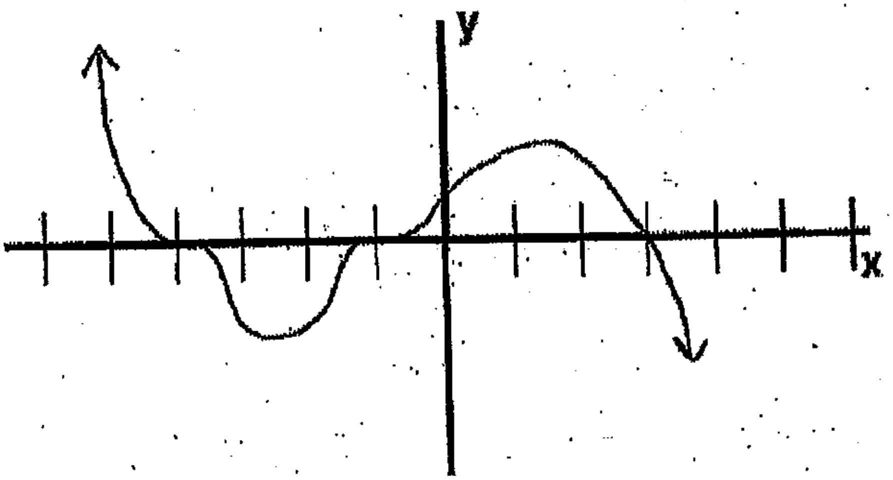
Figure 1.3 **Degree 12**



Zeros: $x = -4, -2, 0, 2, 4, 5$

Factored: $f(x) = -(x+4)^2(x+2)^2(x)(x-2)^3(x-4)^2(x-5)^2$

Figure 1.4 **Degree 7**



Zeros: $x = -4, -1, 3$

Factored: $f(x) = -(x+4)^3(x+1)^3(x-3)$

4.) Find the quotient and remainder when $(x^{11}-4x^9+7x^5-8x^4+11x^3+2x^2-x+10)$ is divided by $(x-2)$. **[5 Points]**

④

$$\begin{array}{r|rrrrrrrrrrrr} 2 & 1 & 0 & -4 & 0 & 0 & 0 & 7 & -8 & 11 & 2 & -1 & 10 \\ & \downarrow & 2 & 4 & 0 & 0 & 0 & 0 & 14 & 12 & 46 & 96 & 190 \\ \hline & 1 & 2 & 0 & 0 & 0 & 0 & 7 & 6 & 23 & 48 & 95 & 200 \\ & x^{10} & x^9 & x^8 & x^7 & x^6 & x^5 & x^4 & x^3 & x^2 & x & C & R \end{array}$$

$$\boxed{\begin{array}{l} q(x)=x^{10}+2x^9+7x^4+6x^3+23x^2+48x+95 \\ r(x)=200. \end{array}}$$

5.) According to the Rational Zeros Theorem, what are all of the possible rational zeros for the function $f(x)=8x^{14}+5x^9-5x^5+13x^3+2x+15$? **[5 Points]**

⑤ $f(x)=\underset{\substack{\downarrow \\ 1,8 \\ 2,4}}{8}x^{14}+5x^9-5x^5+13x^3+2x+\underset{\substack{\downarrow \\ 1,15 \\ 3,5}}{15}$ constant

Biggest power of x (denominators) (numerators)

$$\boxed{\pm\left(1,\frac{1}{2},\frac{1}{4},\frac{1}{8},3,\frac{3}{2},\frac{3}{4},\frac{3}{8},5,\frac{5}{2},\frac{5}{4},\frac{5}{8},15,\frac{15}{2},\frac{15}{4},\frac{15}{8}\right)}$$

6.) Find the value(s) of *k* so that $(x+1)$ is a factor of $5x^3+k^2x^2+2kx-3$. **[5 Points]**

$(x+1)$ is a factor of $f(x)=5x^3+k^2x^2+2kx-3$

$\rightarrow f(-1)=0$

So $f(-1)=5(-1)^3+k^2(-1)^2+2k(-1)-3=0$

$-5+k^2-2k-3=0$

$k^2-2k-8=0$ (or use quadratic formula)

$(k-4)(k+2)=0$

$\boxed{k=4 \text{ or } k=-2}$

7.) Find values of *a* and *b* so that $(x+1)$ and $(x-3)$ are factors of the function $P(x)=x^4-5x^2+ax+b$. **[5 Points]**

$(x+1)$ is a factor of $P(x) \rightarrow P(-1)=0$

$(x-3)$ is a factor of $P(x) \rightarrow P(3)=0$

$0=P(1)=(-1)^4-5(-1)^2+a(-1)+b$

$0=1-5\ a+b$

$0=-4\ a+b \longrightarrow \underline{\underline{-a+b=4}}$

$0=P(3)=3^4-5(3)^2+3a+b$

$0=81-45+3a+b$

$0=36+3a+b \longrightarrow \underline{\underline{3a+b=-36}}$

$$\begin{array}{r} 1 \\ -1 \end{array}\left\{\begin{array}{l} -a+b=4 \\ 3a+b=-36 \end{array}\right.$$

$-a+b=4$

$-3a-b=36$

$-4a=40$

$\boxed{a=-10}$

$-(-10)+b=4$

$\boxed{b=-6}$

$$\boxed{\begin{array}{l} a=-10 \\ b=-6 \end{array}}$$

8.) If three of the zeros of $f(x)=x^4+ax^2+bx+c$ are $x=1$, $x=2$, and $x=3$, what is the value of $b+c$? **[3 Points]**

$$\boxed{24}$$

$x=1$ is a zero $\to f(1)=0$

$1+a+b+c=0$

$\underline{a+b+c=-1}$

$x=2$ is a zero $\to f(2)=0$

$2^4+4a+2b+c=0$

$16+4a+2b+c=0$

$\underline{4a+2b+c=-16}$

$x=3$ is a zero $\to f(3)=0$

$3^4+9a+3b+c=0$

$\underline{9a+3b+c=-81}$

System of Equations!

$$\begin{cases} a+b+c=-1 \\ 4a+2b+c=-16 \\ 9a+3b+c=-81 \end{cases}$$

$$\begin{array}{r|r} & 1 \\ 1 & \\ -1 & -1 \end{array} \begin{cases} 9a+3b+c=-81 \\ 4a+2b+c=-16 \\ a+b+c=-1 \end{cases}$$

$$\begin{array}{r} 4a+2b+c=-16 \\ -a-b-c=1 \\ \hline 3a+b=-15 \end{array}$$

$$\begin{array}{r} 9a+3b+c=-81 \\ -a-b-c=1 \\ \hline 8a+2b=-80 \end{array}$$

$\div 2$

$$\begin{array}{r} -1 \\ 1 \end{array} \begin{cases} 3a+b=-15 \\ 4a+b=-40 \end{cases}$$

$-3a-b=15$

$4a+b=-40$

$\boxed{a=-25}$

$4(-25)+b=-40$

$-100+b=-40$

$\boxed{b=60}$

$-25+60+c=-1$

$35+c=-1$

$\boxed{c=-36}$

So

$b+c=60-36$

$\boxed{b+c=24}$

9.) When a polynomial $P(x)$ is divided by $(x+14)$, the quotient is (x^3-4x+5) and the remainder is -2. **[6 Points]**

A.) Write an expression for $P(x)$ using the divisor, quotient, and remainder form.

B.) Expand your answer in "A" to write the polynomial in standard form.

A.) $\boxed{P(x) = (x+14)(x^3-4x+5) - 2}$

B.) $P(x) = x^4-4x^2+5x+14x^3-56x+70-2$

$\boxed{P(x) = x^4+14x^3-4x^2-51x+68}$

10.) Factor completely: **[12 Points]**

$P(x)=15x^7-22x^6-70x^5+52x^4+123x^3+2x^2-44x-8$.

$P(x) = \underset{\overset{\downarrow}{1,15}\atop 3,5}{15}x^7-22x^6-70x^5+52x^4+123x^3+2x^2-44x-\underset{\overset{\downarrow}{1,8}\atop 2,4}{8}$

Possible Rational Zeros: $\pm\left(1, 2, 4, 8, \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{8}{3}, \frac{1}{5}, \frac{2}{5}, \frac{4}{5}, \frac{8}{5}, \frac{1}{15}, \frac{2}{15}, \frac{4}{15}, \frac{8}{15}\right)$

$P(1) = 48 \neq 0$

$P(-1) = 0$ ✓ $\longrightarrow$ $x=-1$ is a zero / $(x+1)$ is a factor!

-1	15	-22	-70	52	123	2	-44	-8	
	↓	-15	37	33	-85	-38	36	8	
-1	15	-37	-33	85	38	-36	-8	0	☺
	↓	-15	52	-19	-66	28	8		
-1	15	-52	19	66	-28	-8	0		☺
	↓	-15	67	-86	20	8			
-1	$15x^4$	$-67x^3$	$86x^2$	$-20x$	-8 c	0 R			☺
	↓	-15	82	-168	188				
	15	-82	168	-188	180	No!			

So $P(x) = (x+1)^3(15x^4-67x^3+86x^2-20x-8)$

$P(2) = 0$ ✓ $\longrightarrow$ $x=2$ is a zero / $(x-2)$ is a factor!

2	15	-67	86	-20	-8	
	↓	30	-74	24	8	
2	15	-37	12	4	0	☺
	↓	30	-14	-4		
	15	-7	-2	0		☺

$P(x) = (x+1)^3(x-2)^2(15x^2-7x-2)$

$\boxed{P(x) = (x+1)^3(x-2)^2(3x-2)(5x+1)}$

11.) Given that $(1-2i)$ is a zero of **[12 Points]**

$$P(x)=x^7-11x^6+47x^5-105x^4+119x^3+11x^2-167x+105\ ,$$

find all other zeros of $P(x)$ in **exact form**.

$x=1-2i$ is a zero $\rightarrow$ $x=1+2i$ is also a zero (Conjugate Zeros Theorem)

$(x-(1-2i))(x-(1+2i))$ $\rightarrow$ $(x-(1-2i))$ $(x-(1+2i))$ are factors!

$= x^2-(1+2i)x-(1-2i)x+(1-2i)(1+2i)$

$= x^2-x-2ix-x+2ix+1-4i^2$

$= x^2-2x+1-4(-1)$

$= x^2-2x+1+4$

$= \underline{\underline{x^2-2x+5}} \rightarrow$ divides $P(x)$

$$\begin{array}{r|l}
 & x^5-9x^4+24x^3-12x^2-25x+21 \\ \hline
x^2-2x+5 & x^7-11x^6+47x^5-105x^4+119x^3+11x^2-167x+105 \\
 & -(x^7-2x^6+5x^5) \\ \hline
 & -9x^6+42x^5-105x^4+119x^3+11x^2-167x+105 \\
 & -(-9x^6+18x^5-45x^4) \\ \hline
 & 24x^5-60x^4+119x^3+11x^2-167x+105 \\
 & -(24x^5-48x^4+120x^3) \\ \hline
 & -12x^4-x^3+11x^2-167x+105 \\
 & -(-12x^4+24x^3-60x^2) \\ \hline
 & -25x^3+71x^2-167x+105 \\
 & -(-25x^3+50x^2-125x) \\ \hline
 & 21x^2-42x+105 \\
 & -(21x^2-42x+105) \\ \hline
 & 0
\end{array}$$

0 ☺

So

$P(x)=(x^2-2x+5)(\underline{x^5-9x^4+24x^3-12x^2-25x+21})$

Possible Rational Zeros: $\pm(1, 3, 7, 21)$

$P(1)=0$ ✓ $\longrightarrow$ $x=1$ is a zero $\rightarrow (x-1)$ is a factor!

$$\begin{array}{r|rrrrrr}
1 & 1 & -9 & 24 & -12 & -25 & 21 \\
 & \downarrow & 1 & -8 & 16 & 4 & -21 \\ \hline
1 & 1 & -8 & 16 & 4 & -21 & 0 \\
 & \downarrow & 1 & -7 & 9 & 13 & \\ \hline
 & 1 & -7 & 9 & 13 & -8 &
\end{array}$$

0 ☺

No!

$P(x)=(x^2-2x+5)(x-1)(x^4-8x^3+16x^2+4x-21)$ $\rightarrow$ next page

⑪

$\boxed{\text{11 continued}}$

$P(x) = (x^2-2x+5)(x-1)(x^4-8x^3+\ x^2+\ x-\ )$

$P(-1)=0$ ✓ → $x=-1$ is a zero → $(x+1)$ is a factor

$$\begin{array}{r|rrrr|l}
-1 & 1 & -8 & 16 & 4 & -21 \\
 & \downarrow & -1 & 9 & -25 & 21 \\
\hline
-1 & 1 & -9 & 25 & -21 & 0 \ ☺ \\
 & \downarrow & -1 & 10 & -35 & \\
\hline
 & 1 & -10 & 35 & -56 & \text{No!}
\end{array}$$

$P(x) = (x^2-2x+5)(x-1)(x+1)(x^3-9x^2+25x-21)$

$P(3)=0 \rightarrow x=3$ is a zero → $(x-3)$ is a factor!

$$\begin{array}{r|rrrr}
3 & 1 & -9 & 25 & -21 \\
 & \downarrow & 3 & -18 & 21 \\
\hline
 & 1 & -6 & 7 & 0 \ ☺
\end{array}$$

$P(x) = (x^2-2x+5)(x-1)(x+1)(x-3)(x^2-6x+7)$

$x=1-2i$
$x=1+2i$

$x=1$

$x=-1$

$x=3$

$x = \dfrac{6 \pm \sqrt{36-28}}{2}$

$x = \dfrac{6 \pm \sqrt{8}}{2}$

$x = \dfrac{6 \pm 2\sqrt{2}}{2}$

$x = 3 \pm \sqrt{2}$

Other Zeros:

$x=1+2i$
$x=1$
$x=-1$
$x=3$
$x=3+\sqrt{2}$
$x=3-\sqrt{2}$

12.) Solve the equation for all complex values of x (**in exact form**) given that **[12 Points]**

$x = -1 + \sqrt{2}$ is a solution:

$$2x^8 + x^7 + 105x^5 + 78x^4 + 202x^2 + 107x = 14x^6 + 429x^3 + 52$$

⑫ $2x^8+x^7+105x^5+78x^4+202x^2+107x = 14x^6+429x^3+52$

Set $P(x) = 2x^8+x^7-14x^6+105x^5+78x^4-429x^3+202x^2+107x-52 = 0$

$x=-1+\sqrt{2}$ is a solution $\rightarrow$ $x=-1-\sqrt{2}$ is also a solution.

(conjugate Zeros Theorem)

$\rightarrow (x-(-1+\sqrt{2}))$
$(x-(-1-\sqrt{2}))$ are factors

$(x-(-1+\sqrt{2}))(x-(-1-\sqrt{2}))$
$= x^2-(-1-\sqrt{2})x-(-1+\sqrt{2})x+(-1+\sqrt{2})(-1-\sqrt{2})$
$= x^2+x+\sqrt{2}x+x-\sqrt{2}x+1-2$
$= x^2+2x-1 \longrightarrow$ divides $P(x)$

$$
\begin{array}{r}
2x^6-3x^5-6x^4+114x^3-156x^2-3x+52 \\
x^2+2x-1 \,\big)\, 2x^8+x^7-14x^6+105x^5+78x^4-429x^3+202x^2+107x-52 \\
-(2x^8+4x^7-2x^6) \\
\hline
-3x^7-12x^6+105x^5+78x^4-429x^3+202x^2+107x-52 \\
-(-3x^7-6x^6+3x^5) \\
\hline
-6x^6+102x^5+78x^4-429x^3+202x^2+107x-52 \\
-(-6x^4-12x^5+6x^4) \\
\hline
114x^5+72x^4-429x^3+202x^2+107x-52 \\
-(114x^5+228x^4-114x^3) \\
\hline
-156x^4-315x^3+202x^2+107x-52 \\
-(-156x^4-312x^3+156x^2) \\
\hline
-3x^3+46x^2+107x-52 \\
-(-3x^3-6x^2+3x) \\
\hline
52x^2+104x-52 \\
-(52x^2+104x-52) \\
\hline
0
\end{array}
$$

☺

So

$P(x) = (x^2+2x-1)(2x^6-3x^5-6x^4+114x^3-156x^2-3x+52)$

1, 2

1, 52
2, 26
4, 13

Possible Rational Zeros =
$\pm(1, 2, 4, 13, 26, 52, \frac{1}{2}, \frac{13}{2})$

$P(1)=0 \rightarrow x=1$ is a zero $\rightarrow x-1$ is a factor

1	2	-3	-6	114	-156	-3	52	
	↓	2	-1	-7	107	-49	-52	
1	2	-1	-7	107	-49	-52		0 ☺
	↓	2	1	-6	101	52		
1	2	1	-6	101	52	0		☺
	↓	2	3	-3	98			
	2	3	-3	98	No			

next page

⑫

[12 continued]

So

$P(x) = (x^2+2x-1)(x-1)^2(2x^4+x^3-6x^2+101x+52)$

$P(-1) = 432 \neq 0$ So $x=-1$ is <u>NOT</u> a zero

$P(2) = 1890 \neq 0$ So $x=2$ is <u>NOT</u> a zero

$P(-2) = 1350 \neq 0$ So $x=-2$ is <u>NOT</u> a zero

$P(4) = 113752 \neq 0$ So $x=4$ is <u>NOT</u> a zero

$P(-4) = 0$ So $x=-4$ is a zero → $(x+4)$ is a factor

$$\begin{array}{r|rrrr|r}
-4 & 2 & 1 & -6 & 101 & 52 \\
 & \downarrow & -8 & 28 & -88 & -52 \\
\hline
-4 & 2 & -7 & 22 & 13 & 0 \ ☺ \\
 & \downarrow & -8 & 60 & \text{No!} & \\
\hline
 & 2 & -15 & 82 & &
\end{array}$$

$P(x) = (x^2+2x-1)(x-1)^2(x+4)(2x^3-7x^2+22x+13)$

$\overset{\downarrow}{1,2}$ $\qquad$ $\overset{\downarrow}{1,13}$

Possible Rational Zeros
$\pm\left(1, 13, \frac{1}{2}, \frac{13}{2}\right)$

$P(13) \neq 0$

$P(-13) \neq 0$

$P(\frac{1}{2}) \neq 0$

$P(-\frac{1}{2}) = 0$ ✓ → $x = -\frac{1}{2}$ is a zero $(2x+1)$ is a factor

$$\begin{array}{r|rrr|r}
-\frac{1}{2} & 2 & -7 & 22 & 13 \\
 & \downarrow & -1 & 4 & -13 \\
\hline
 & \frac{2}{2} & \frac{-8}{2} & \frac{26}{2} & 0 \ ☺ \\
 & 1 & -4 & 13 &
\end{array}$$

$P(x) = (x^2+2x-1)(x-1)^2(x+4)(2x+1)(x^2-4x+13) = 0$

$x = -1+\sqrt{2}$ $\quad$ $x=1$ $\quad$ $x=-4$ $\quad$ $x = -\frac{1}{2}$ $\quad$ $x = \frac{4 \pm \sqrt{16-52}}{2}$

$x = -1-\sqrt{2}$

$x = \frac{4 \pm \sqrt{-36}}{2}$

$x = \frac{4 \pm 6i}{2}$

$x = 2 \pm 3i$

All complex solutions:

$x = -1+\sqrt{2}$ $\quad$ $x=1$ $\quad$ $x = -\frac{1}{2}$

$x = -1-\sqrt{2}$ $\quad$ $x = -4$

$x = 2+3i$ $\quad$ $x = 2-3i$

13.) Sketch a graph of **[12 Points]**

$f(x)=-2x^6+4x^5+16x^4-28x^3-22x^2+56x-24$

by hand. This includes justifying all work used to render your graph.

You cannot simply view this on your graphing calculator for credit!!

$f(x)=-2x^6+4x^5+16x^4-28x^3-22x^2+56x-24$

$f(x)=-2(x^6-2x^5-8x^4+14x^3+11x^2-28x+12)$

Possible Rational Zeros: ±(1, 2, 3, 4, 6, 12)

$f(1)=0 \rightarrow x=1$ is a zero $\rightarrow (x-1)$ is a factor

1	1	-2	-8	14	11	-28	12	
	↓	1	-1	-9	5	16	-12	
1	1	-1	-9	5	16	-12	0	☺
	↓	1	0	-9	-4	12		
1	1	0	-9	-4	12	0		☺
	↓	1	1	-8	-12			
1	1	1	-8	-12	0			☺
	↓	1	2	-6				
	1	2	-6	-18				No!

$f(x)=-2(x-1)^3(x^3+x^2-8x-12)$

$f(-1)=-64\neq 0$

$f(2)=32\neq 0$

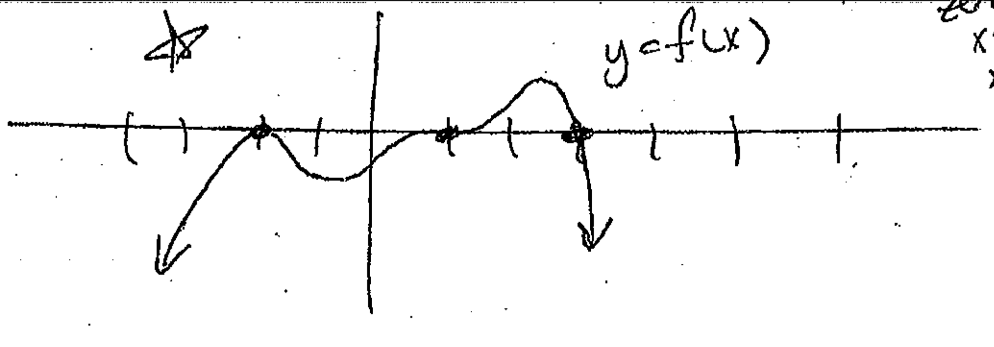
$f(-2)=0 \rightarrow x=-2$ is a zero $\rightarrow (x-2)$ is a factor!

-2	1	1	-8	-12	
	↓	-2	2	12	
	1	-1	-6	0	☺

$f(x)=-2(x-1)^3(x+2)(x^2-x-6)$

$\underline{f(x)=-2(x-1)^3(X+2)(x-3)(x+2)}$

$\boxed{f(x)=-2(x-1)^3(x+2)^2(x-3)}$ Degree 6 EB ↓ ↓



Zeros
$x=1 \sim x^3$
$x=-2 \sim x^2$
$x=3 \sim x$

14.) Suppose *P(x)* is a polynomial constructed such that when it is divided by $(x-20)$ **[2 Points]** the remainder is 12 and when it is divided by $(x-12)$ the remainder is 20. Determine the remainder when *P(x)* is divided by $(x-20)(x-12)$.

When P(x) is divided by

$(x-20)(x-12) = x^2-20x-12x+240$,

its remainder must have degree less than 2.

Answer: $\boxed{r(x) = -x+32}$

So

$r(x) = Ax+B$, when $\frac{P(x)}{(x-20)(x-12)}$.

Now since $\frac{P(x)}{x-20}$ has remainder 12, $P(20) = 12$,

and since $\frac{P(x)}{x-12}$ has remainder 20, $P(12) = 20$,

When $\frac{P(x)}{(x-20)(x-12)}$, there is a quotient $q(x)$ so

$P(x) = q(x)(x-20)(x-12) + r(x)$.

So $P(20) = r(20) = A(20)+B = 12$

and $P(12) = r(12) = A(12)+B = 20$

So $\begin{matrix} 1 \\ -1 \end{matrix}\begin{cases} 20A+B=12 \\ 12A+B=20 \end{cases}$

$$\begin{array}{r} 20A+B=12 \\ -12A-B=-20 \\ \hline 8A\quad=-8 \\ \boxed{A=-1} \end{array}$$

$-20+B=12$

$\boxed{B=32}$

So $r(x) = Ax+B$

$\boxed{r(x) = -x+32}$