

Advanced Precalculus HW Key (2/16/12)

① $x^4 - 3x^3 + x^2 + 3x - 2 > 0$

Set $P(x) = x^4 - 3x^3 + x^2 + 3x - 2$

Possible Rational Zeros: $\pm(1, 2)$

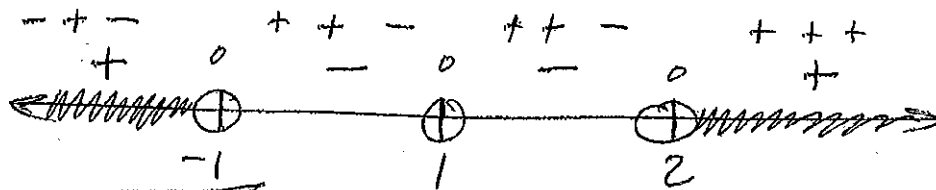
$P(1) = 0 \checkmark \rightarrow x=1$ is a zero $\rightarrow (x-1)$ is a factor

1	1	-3	1	3	-2	
	↓		-2	-1	2	
1	1	-2	-1	2	0	II
	↓		-1	-2		
1	1	-1	-2	0		II
	↓		1	0		
	1	0	No!			

$P(x) = (x-1)^2(x^2-x-2)$

$P(x) = (x-1)^2(x-2)(x+1)$

$P(x) = (x+1)(x-1)^2(x-2) > 0$



$(-\infty, -1) \cup (2, \infty)$

②

$$36x^4 - 13x^2 + 1 < 0$$

$$\text{Set } P(x) = 36x^4 - 13x^2 + 1$$

Possible Rational zeros: $\pm (1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \frac{1}{18}, \frac{1}{36})$

$$P(1) = 24 \neq 0 \text{ so } x=1 \text{ is not a zero}$$

$$P(-1) = +24 \neq 0 \text{ so } x=-1 \text{ is not a zero}$$

$$P(\frac{1}{2}) = 0 \text{ so } x = \frac{1}{2} \text{ is a zero} \rightarrow (2x-1) \text{ is a factor}$$

$\frac{1}{2}$	36	0	-13	0	1	
	↓	18	9	-2	-1	
$\frac{1}{2}$	36	18	-4	-2	0	⊥
	↓	18	18	1		
$\frac{1}{2}$	36	36	14	5	-1	No!
	↓	18	9	-2	-1	

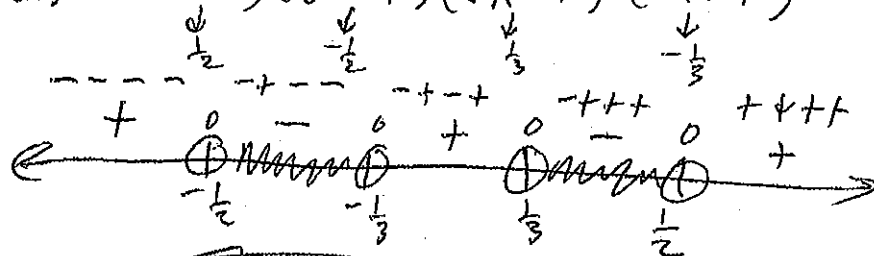
$$P(x) = (2x-1)(18x^3 + 9x^2 - 2x - 1)$$

$$P(-\frac{1}{2}) = 0 \text{ so } x = -\frac{1}{2} \text{ is a zero} \rightarrow (2x+1) \text{ is a factor}$$

$-\frac{1}{2}$	18	9	-2	-1	
	↓	-9	0	1	
	18	0	-2	0	⊥
	$\frac{18}{2}$	9	$\frac{-2}{2}$	-1	

$$P(x) = (2x-1)(2x+1)(9x^2-1)$$

$$P(x) = (2x-1)(2x+1)(3x-1)(3x+1) < 0$$



$$\left(-\frac{1}{2}, -\frac{1}{3}\right) \cup \left(\frac{1}{3}, \frac{1}{2}\right)$$

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$$4x^5 + 27x^4 + 65x^3 + 62x^2 + 12x - 8 \geq 0$$

Set $P(x) = 4x^5 + 27x^4 + 65x^3 + 62x^2 + 12x - 8$

Possible Rational Zeros: $\pm (1, 2, 4, 8, \frac{1}{2}, \frac{1}{4})$

$P(1) = 162$ so $x=1$ is not a zero
 $P(-1) = 0$ so $x=-1$ is a zero $\rightarrow (x+1)$ is a factor

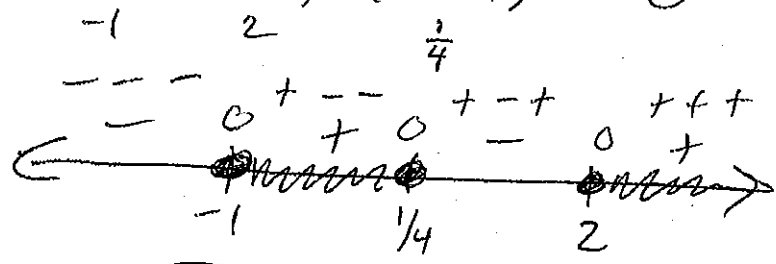
-1	4	27	65	62	12	-8	
	↓	-4	-23	-42	-20	8	
-1	4	23	42	20	-8	0	☺
	↓	-4	-11	-23	3		
	4	17	23	-3	No!		

$P(x) = (x+1)(4x^4 + 23x^3 + 42x^2 + 20x - 8)$

$P(2) = 1344$ so $x=2$ is not a zero
 $P(-2) = 0$ so $x=-2$ is a zero $\rightarrow (x+2)$ is a factor

-2	4	23	42	20	-8	
	↓	-8	-30	-24	8	
-2	4	15	12	-4	0	☺
	↓	-8	-14	4		
-2	4	7	-2	0	☺	
	↓	-8	2			
	4	-1	0	☺		

$P(x) = (x+1)(x-2)^3(4x-1) \geq 0$



$$[-1, \frac{1}{4}] \cup [2, \infty)$$

④ $3x^5 - x^4 + 11x^3 - x^2 - 42x + 30 < 0$

Set $P(x) = 3x^5 - x^4 + 11x^3 - x^2 - 42x + 30$

Possible Rational Zeros: $\pm(1, 2, 3, 5, 6, 10, 15, 30, \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{10}{3})$

$P(1) = 0 \rightarrow x=1$ is a zero $\rightarrow (x-1)$ is a factor

1	3	-1	11	-1	-42	30	
	↓	3	2	13	12	-30	
1	3	2	13	12	-30	0	"
	↓	3	5	18	30		
1	3	5	18	30	0		"
	↓	3	8	26			
	3	8	26				No!

$P(x) = (x-1)^2 (3x^3 + 5x^2 + 18x + 30)$

$P(-1) = 56$ so $x=-1$ is not a zero

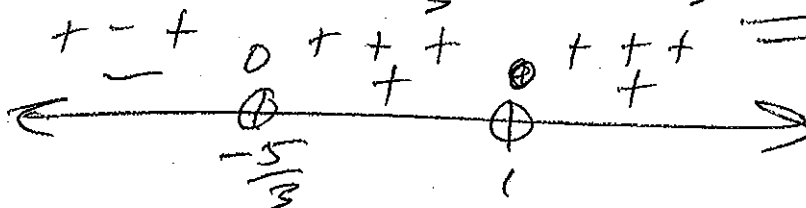
check

$P(-\frac{5}{3}) = 0$ so $(3x+5)$ is a factor

$-\frac{5}{3}$	3	5	18	30
	↓	-5	0	-30
$\frac{5}{3}$	3	0	$\frac{18}{3}$	0
			6	

$P(x) = (x-1)^2 (3x+5)(x^2+6) < 0$ $x^2+6=0$

\rightarrow no real solutions



$(-\infty, -\frac{5}{3})$

5) Set $P(x) = 12x^9 + 13x^8 - 36x^7 - 40x^6 + 36x^5 + 42x^4 - 12x^3 - 16x^2 + 1 > 0$

$P(1) = 0 \rightarrow x=1$ is a zero $\rightarrow (x-1)$ is a factor

1	12	13	-36	-40	36	42	-12	-16	0	1	
	↓	12	25	-11	-51	-15	27	15	-1	-1	
1	12	25	-11	-51	-15	27	15	-1	-1	0	✓
	↓	12	37	26	-25	-40	-13	2	1	0	
1	12	37	26	-25	-40	73	2	1	0	0	✓
	↓	12	49	75	50	10	-3	-1	0	0	✓
1	12	49	75	50	10	-3	-1	0	0	0	✓
	↓	12	61	136	186						✓
	12	61	136	186	196						No

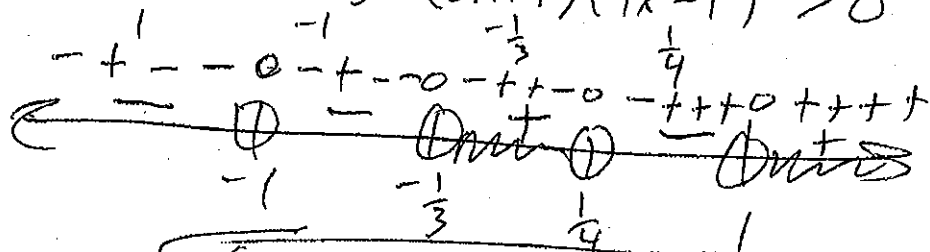
$P(x) = (x-1)^3 (12x^6 + 49x^5 + 75x^4 + 50x^3 + 10x^2 - 3x - 1)$

$P(-1) = 0 \rightarrow x=-1$ is a zero $\rightarrow (x+1)$ is a factor

-1	12	49	75	50	10	-3	-1	
	↓	-12	-37	-38	-12	2	1	
-1	12	37	38	12	-2	-1	0	✓
	↓	-12	-25	-13	1	1	0	✓
-1	12	25	13	-1	-1	0	0	✓
	↓	-12	-13	0	1	0	0	✓
-1	12	13	0	-1	0	0	0	✓
	↓	-12	-1	1	0	0	0	✓
-1	12	1	-1	0	0	0	0	✓
	↓	-12	11	0	0	0	0	✓
	12	-11	10					No!

$P(x) = (x-1)^3 (x+1)^4 (12x^2 + x - 1)$

$P(x) = (x-1)^3 (x+1)^4 (3x+1)(4x-1) > 0$



$(-\frac{1}{3}, \frac{1}{4}) \cup (1, \infty)$

⑥ $x = \sqrt{7}$ is a solution
 $\rightarrow x = -\sqrt{7}$ is a solution
 (Conjugate Zeros Theorem)

$$(x - \sqrt{7})(x + \sqrt{7}) = x^2 - 7 \text{ is a factor...}$$

$$x^7 + 3x^6 - 7x^5 - 26x^4 - x^3 + 37x^2 + 7x - 14 = 0$$

$$\begin{array}{r}
 x^5 + 3x^4 \quad -5x^2 - x + 2 \\
 x^2 + 0x - 7 \overline{) x^7 + 3x^6 - 7x^5 - 26x^4 - x^3 + 37x^2 + 7x - 14} \\
 \underline{-(x^7 \quad -7x^5)} \\
 3x^6 + 0x^5 - 26x^4 - x^3 + 37x^2 + 7x - 14 \\
 \underline{-(3x^6 + 0x^5 - 21x^4)} \\
 -5x^4 - x^3 + 37x^2 + 7x - 14 \\
 \underline{-(-5x^4 + 0x^3 + 35x^2)} \\
 -x^3 + 2x^2 + 7x - 14 \\
 \underline{-(-x^3 + 0x^2 + 7x)} \\
 2x^2 + 0x - 14 \\
 \underline{-(2x^2 + 0x - 14)} \\
 0 \quad \text{||}
 \end{array}$$

Quotient: $x^5 + 3x^4 - 5x^2 - x + 2$

Possible Rational Zeros: $\pm(1, 2)$

$$x=1: 1 + 3 - 5 - 1 + 2 = 0 \checkmark$$

$x=1$ is a zero
 $(x-1)$ is a factor

$x=-1$ is a zero
 $(x+1)$ is a factor
 $-1 + 3 - 5 + 1 + 2 = 0$

$x=-2$ is a zero
 $(x+2)$ is a factor

1	1	3	0	-5	-1	2	
	↓	1	4	4	-1	-2	
-1	1	4	4	-1	-2	0	
	↓	-1	-3	-1	2		
-2	1	3	1	-2	0	0	
	↓	-2	-2	2			
	1	1	-1	0			

$x=1$ doesn't work twice

$x=-1$ doesn't work twice

So...

$$(x^2 - 7)(x+1)(x-1)(x+2)(x^2 + x - 1) = 0$$

$x = \sqrt{7}$	$x = -\sqrt{7}$	$x = -1$	$x = 1$	$x = -2$	$x = \frac{-1 \pm \sqrt{1+4}}{2}$
$x = -\sqrt{7}$					$x = \frac{-1 \pm \sqrt{5}}{2}$

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$x=i$ is a solution
So $x=-i$ is a solution

$$\begin{aligned} &\rightarrow (x-i)(x+i) \\ &= x^2 - i^2 \\ &= x^2 - (-1) \\ &= x^2 + 1 \end{aligned}$$

factor

$$\begin{array}{r} x^6 - 4x^4 - 6x^3 - 7x^2 - 6x - 2 = 0 \\ x^4 + 0x^3 - 5x^2 - 6x - 2 \\ \hline x^2 + 0x + 1 \cdot \left\{ \begin{array}{l} x^6 + 0x^5 - 4x^4 - 6x^3 - 7x^2 - 6x - 2 \\ -(x^6 + 0x^5 + x^4) \\ \hline -5x^4 - 6x^3 - 7x^2 - 6x - 2 \\ -(-5x^4 + 0x^3 - 5x^2) \\ \hline -6x^3 - 2x^2 - 6x - 2 \\ -(-6x^3 + 0x^2 - 6x) \\ \hline -2x^2 + 0x - 2 \\ -(-2x^2 + 0x - 2) \\ \hline 0 \end{array} \right. \end{array}$$

So $(x^2+1)(x^4-5x^2-6x-2)=0$
 \rightarrow Possible Rational Zeros: $\pm(1, 2)$
 $x=1: 1-5-6-2 \neq 0$
 $x=-1: 1-5+6-2 = 0 \checkmark$

$x=-1$ is a zero
 $(x+1)$ is a factor

$$\begin{array}{r|rrrrr} -1 & 1 & 0 & -5 & -6 & -2 \\ & \downarrow & -1 & 1 & 4 & 2 \\ -1 & 1 & -1 & -4 & -2 & 0 \\ & \downarrow & -1 & 2 & 2 & \\ & 1 & -2 & -2 & 0 & \end{array}$$

$$(x^2+1)(x+1)^2(x^2-2x-2) = 0$$

$x=i$ $x=-1$ $x=1 \pm \sqrt{3}$
 $x=-i$ $x=-1$ $x=1 \pm \sqrt{3}$

$$x = \frac{2 \pm \sqrt{4+8}}{2}$$

$$x = \frac{2 \pm \sqrt{12}}{2}$$

$$x = \frac{2 \pm 2\sqrt{3}}{2}$$

$$x = 1 \pm \sqrt{3}$$