

Advanced Precalculus : HW Key 2/13/12

$$\begin{aligned} \textcircled{1} \quad p(-1) &= 7(-1)^4 - 5(-1)^3 + 3(-1)^2 - 5 \\ &= -7 + 5 + 3 - 5 \\ &= -4 \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad p(-1) &= (-1)^{60} - (-1)^{10} - (-1)^2 - 1 \\ &= 1 - 1 - 1 - 1 \\ &= -2 \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad p(1) &= 18(1)^{22} - 5(1)^{17} + 4(1)^3 - 8(1)^2 + 1 \\ &= 18 - 5 + 4 - 8 + 1 \\ &= 13 - 4 + 1 \\ &= 10 \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad p(-2) &= 8(-2)^5 - (-2)^4 - (-2)^3 - (-2)^2 - 3(-2) + 1 \\ &= 8(-32) - (16) - (-8) - (4) + 6 + 1 \\ &= -256 - 16 + 8 - 4 + 6 + 1 \\ &= -261 \end{aligned}$$

$$\begin{aligned} \textcircled{5} \quad p(3) &= 3(3)^4 - (3)^2 - 8(3) + 2 \\ &= 243 - 9 - 24 + 2 \\ &= 212 \end{aligned}$$

$$\begin{aligned} \textcircled{12} \quad f(x) &= 3x^4 - 20x^3 + 41x^2 - 20x - 12 \\ &\quad \downarrow \qquad \qquad \qquad \downarrow \\ &\quad 1, 3 \qquad \qquad \qquad 1, 2, 4, 6, 12 \\ &\pm \left(1, 2, 3, 4, 6, 12, \frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{6}{3}, \frac{12}{3} \right) \\ &= \pm \left(1, 2, 3, 4, 6, 12, \frac{1}{3}, \frac{2}{3}, \frac{4}{3} \right) \end{aligned}$$

$$\begin{aligned} \textcircled{13} \quad g(x) &= 2x^3 - 2x + 8 \\ &\quad \downarrow \qquad \qquad \qquad \downarrow \\ &\quad 1, 2 \qquad \qquad \qquad 1, 2, 4, 8 \\ &\pm \left(1, 2, 4, 8, \frac{1}{2}, \frac{2}{2}, \frac{4}{2}, \frac{8}{2} \right) \\ &= \pm \left(1, 2, 4, 8, \frac{1}{2} \right) \end{aligned}$$

$$\begin{aligned} \textcircled{6} \quad f(2) &= 3(2)^4 - 20(2)^3 + 41(2)^2 - 20(2) - 12 \\ &= 3(16) - 20(8) + 41(4) - 40 - 12 \\ &= 48 - 160 + 164 - 40 - 12 \\ &= 0 \end{aligned}$$

Remainder 0 → Yes a factor!

$$\begin{aligned} \textcircled{7} \quad g(-1) &= (-1)^{60} - 1 \\ &= 1 - 1 \\ &= 0 \end{aligned}$$

Remainder 0 → Yes a factor!

$$\begin{aligned} \textcircled{8} \quad f(1) &= 1^{48} - 2 \\ &= 1 - 2 \\ &= -1 \end{aligned}$$

Remainder -1 → Not a factor!

$$\begin{aligned} \textcircled{9} \quad g(5) &= 5^5 - 5 - 5^4 - 81(5) + 405 \\ &= 3125 - 3125 - 405 + 405 \\ &= 0 \end{aligned}$$

Remainder 0 → Yes a factor!

$$\begin{aligned} \textcircled{10} \quad f(-1) &= (-1)^{11} - (-1)^3 + 1 \\ &= -1 - (-1) + 1 \\ &= -1 + 1 + 1 \\ &= 1 \end{aligned}$$

Remainder 1 → Not a factor!

$$\begin{aligned} \textcircled{11} \quad g(-2) &= 9(-2)^3 - 3(-2)^2 + (-2) + 1 \\ &= 9(-8) - 3(4) - 2 + 1 \\ &= -72 - 12 - 2 + 1 \\ &= -85 \end{aligned}$$

Remainder -85 → Not a factor!

(14) $f(x) = 6x^3 - 2x^2 + 3x - 1$

\downarrow \downarrow
 $\frac{1}{2}, \frac{1}{3}$ $\frac{1}{6}$

$\pm \left(-1, \frac{1}{2}, \frac{1}{3}, \frac{1}{6} \right)$

(15) $g(x) = 8x^2 + 2x - 15$

\downarrow \downarrow
 $\frac{1}{2}, \frac{1}{4}$ $\frac{1}{15}, \frac{3}{5}$

$\pm \left(1, 3, 5, 15, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{15}{2}, \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{15}{4}, \frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{15}{8} \right)$

(16) $p(2) = 3(2)^3 - 4(2)^2 + k(2) + 2 = 0$

$24 - 16 + 2k + 2 = 0$
 $2k + 10 = 0$
 $2k = -10$
 $k = -5$

$k = -5$

(17) $p(-1) = (-1)^3 + k(-1)^2 - 5(-1) - 3 = 0$

$-1 + k + 5 - 3 = 0$
 $k + 1 = 0$
 $k = -1$

$k = -1$

(18) $p(1) = 4 + A + B - 3 = 0 \rightarrow A + B = -1$

$p(-3) = 4(-3)^3 + A(-3)^2 + B(-3) - 3 = 0$
 $-108 + 9A - 3B - 3 = 0$
 $9A - 3B = 111$

$\begin{cases} 1 & 9A - 3B = 111 \\ 3 & A + B = -1 \end{cases}$
 $9A - 3B = 111$
 $3A + 3B = -3$
 $\hline 12A = 108 \rightarrow A = 9$

$A + B = -1$
 $B = -10$

$A = 9$
 $B = -10$

① $f(x) = x^3 - 10x^2 + 17x + 28$

Possible Rational Zeros: $\pm (1, 2, 4, 7, 14, 28)$

$f(1) = 36 \neq 0$
 $f(-1) = 0 \checkmark$

$(x+1)$ is a factor

$$\begin{array}{r|rrrr} -1 & 1 & -10 & 17 & 28 \\ & \downarrow & -1 & 11 & -28 \\ \hline & 1 & -11 & 28 & 0 \end{array}$$

$x^2 \quad x \quad c \quad R$

$f(x) = (x+1)(x^2 - 11x + 28)$

$f(x) = (x+1)(x-7)(x-4)$

② $f(x) = 6x^4 + 5x^3 - 23x^2 - 20x - 4$

Possible Rational Zeros: $\pm (1, \frac{1}{2}, \frac{1}{3}, \frac{1}{6}, 2, \frac{2}{3}, 4, \frac{4}{3})$

$f(1) = -36$
 $f(-1) = -6$
 $f(2) = 0 \checkmark$

$(x-2)$ is a factor

$f(-2) = 0 \checkmark$

$(x+2)$ is a factor

$$\begin{array}{r|rrrrr} 2 & 6 & 5 & -23 & -20 & -4 \\ & \downarrow & 12 & 34 & 22 & 4 \\ \hline 2 & 6 & 17 & 11 & 2 & 0 \\ & \downarrow & 12 & 58 & 138 & \\ \hline & 6 & 29 & 69 & \text{No!} & \end{array}$$

again?

$$\begin{array}{r|rrrr} -2 & 6 & 17 & 11 & 2 \\ & \downarrow & -12 & -10 & -2 \\ \hline & 6 & 5 & 1 & 0 \\ & x^2 & x & c & R \end{array}$$

$f(x) = (x-2)(x+2)(6x^2 + 5x + 1)$

$f(x) = (x-2)(x+2)(3x+1)(2x+1)$

or

$6x^2 + 5x + 1$
 $= 6x^2 + 3x + 2x + 1$
 $= 3x(2x+1) + 1(2x+1)$
 $= (2x+1)(3x+1)$

*6
+5
+3,+2