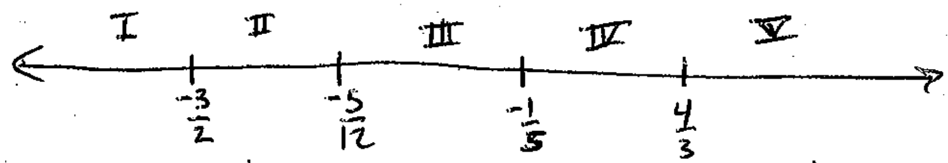
WA #1 Key

1.) Find all solutions to the equation below by hand. **[7 Points]**
(You cannot have a calculator solve it for you.)

$$\left|x-|5x+1|+|2x+3|\right|+|3x-4|=|12x+5|$$



Case I: $x \le -\frac{3}{2}$:

$|x+5x+1-(2x+3)| -3x+4 = -12x-5$

$|x+5x+1-2x-3| -3x+4 = -12x-5$

$|4x-2| -3x+4 = -12x-5$

Key #: $\frac{1}{2}$ Not in case

$-(4x-2)-3x+4 = -12x-5$

$-4x+2-3x+4 = -12x-5$

$-7x+6 = -12x-5$

$5x = -11$

$x = -\frac{11}{5}$

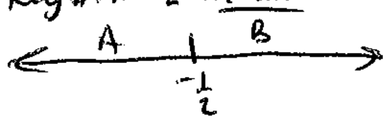
yes!

Case II: $-\frac{3}{2} \le x \le -\frac{5}{12}$

$|x+5x+1+2x+3| -3x+4 = -12x-5$

$|8x+4| -3x+4 = -12x-5$

Key #: $x = -\frac{1}{2}$ in case



Case A: $x \le -\frac{1}{2}$:

$-8x-4-3x+4 = -12x-5$

$-11x = -12x-5$

$x = -5$

No! (Doesn't work for Case II)

Case B: $x \ge -\frac{1}{2}$:

$8x+4-3x+4 = -12x-5$

$5x+8 = -12x-5$

$17x = -13$

$x = -\frac{13}{17}$

No! (Doesn't work for Case B)

Case III: $-\frac{5}{12} \le x \le -\frac{1}{5}$

$|x+5x+1+2x+3| -3x+4 = 12x+5$

$|8x+4| -3x+4 = 12x+5$

Key #: $-\frac{1}{2}$ NOT in case

$8x+4-3x+4 = 12x+5$

$5x+8 = 12x+5$

$3 = 7x$

$x = \frac{3}{7}$

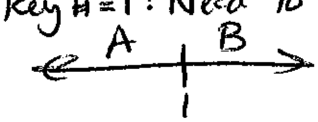
Not a solution (Doesn't work in Case III)

Case IV: $-\frac{1}{5} \le x \le \frac{4}{3}$

$|x-5x-1+2x+3| -3x+4 = 12x+5$

$|-2x+2| -3x+4 = 12x+5$

Key # = 1 : Need to split



Case A: $x \le 1$

$-2x+2-3x+4 = 12x+5$

$-5x+6 = 12x+5$

$1 = 17x$

$x = \frac{1}{17}$ yes!

Case B: $x \ge 1$

$2x-2-3x+4 = 12x+5$

$-x+2 = 12x+5$

$-3 = 13x$

$x = -\frac{3}{13}$ No!

Not in case B

Case V: $x \ge \frac{4}{3}$

$|x-5x-1+2x+3| +3x-4 = 12x+5$

$|-2x+2| +3x-4 = 12x+5$

Key # = 1 Not in case

$2x-2+3x-4 = 12x+5$

$5x-6 = 12x+5$

$-11 = 7x$

$-\frac{11}{7} = x$

No!
Not in case V

★ **Overall:**

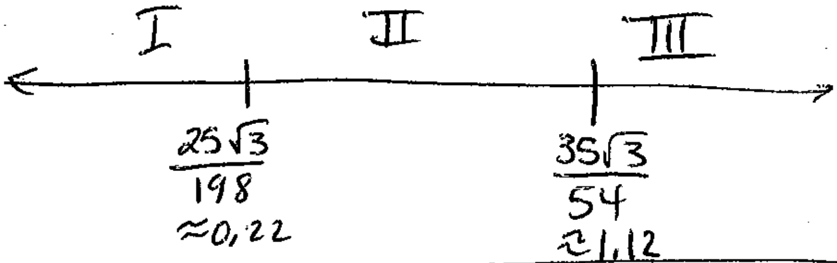
$x = -\frac{11}{5}$

$x = \frac{1}{17}$

2.) Find all solutions to the equation below by hand. (You cannot have a calculator solve it for you.) **[7 Points]**

$$\left|\frac{\sqrt{27}x}{5}-\frac{7}{6}\right|+\frac{9}{2}=\left|\frac{5}{12}-\frac{\sqrt{363}x}{10}\right|$$

$$\left|\frac{3\sqrt{3}}{5}x-\frac{7}{6}\right|+\frac{9}{2}=\left|\frac{5}{12}-\frac{11\sqrt{3}}{10}x\right|$$



Key #5:

$\frac{3\sqrt{3}}{5}x=\frac{7}{6}$

$x=\frac{35}{18\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{35\sqrt{3}}{54}$

≈ 1.12

$\frac{11\sqrt{3}}{10}x=\frac{5}{12}$

$x=\frac{50}{12(11)\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$

$x=\frac{50\sqrt{3}}{12(11)(3)}$

$x=\frac{25\sqrt{3}}{198}$

≈ 0.22

Case I: $x\le\frac{25\sqrt{3}}{198}$

$60\left(-\frac{3\sqrt{3}}{5}x+\frac{7}{6}+\frac{9}{2}\right)=\left(\frac{5}{12}-\frac{11\sqrt{3}}{10}x\right)60$

$-36\sqrt{3}x+70+270=25-66\sqrt{3}x$

$30\sqrt{3}x+340=25$

$30\sqrt{3}x=-315$

$x=\frac{-315}{30\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$

$x=\frac{-315\sqrt{3}}{30(3)}$

$x=\frac{-35\sqrt{3}}{10}$

$\boxed{x=\frac{-7\sqrt{3}}{2}}$

Yes!

Case II: $\frac{25\sqrt{3}}{198}\le x\le\frac{35\sqrt{3}}{54}$

$60\left(-\frac{3\sqrt{3}}{5}x+\frac{7}{6}+\frac{9}{2}\right)=\left(-\frac{5}{12}+\frac{11\sqrt{3}}{10}x\right)60$

$-36\sqrt{3}x+70+270=-25+66\sqrt{3}x$

$340=-25+102\sqrt{3}x$

$365=102\sqrt{3}x$

$x=\frac{365}{102\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$

$x=\frac{365\sqrt{3}}{102(3)}$

$x=\frac{365\sqrt{3}}{306}\approx 2.06$

No! Not in Case II

Case III: $x\ge\frac{35\sqrt{3}}{54}$

$60\left(\frac{3\sqrt{3}}{5}x-\frac{7}{6}+\frac{9}{2}\right)=\left(-\frac{5}{12}+\frac{11\sqrt{3}}{10}x\right)60$

$36\sqrt{3}x-70+270=-25+66\sqrt{3}x$

$36\sqrt{3}x+200=-25+66\sqrt{3}x$

$225=30\sqrt{3}x$

$x=\frac{225}{30\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$

$x=\frac{225\sqrt{3}}{90}$

$\boxed{x=\frac{5\sqrt{3}}{2}}$

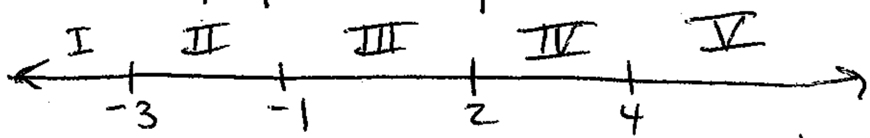
Yes!

$$\boxed{\text{Overall: } x=\frac{-7\sqrt{3}}{2},\quad x=\frac{5\sqrt{3}}{2}}$$

3.) Find all solutions to the equation below by hand. (You cannot have a calculator solve it for you.) **[7 Points]**

$$|x^2+x-6| = |x^2-3x-4|+2$$

$$|(x+3)(x-2)| = |(x-4)(x+1)| + 2$$



Case I: $x \le -3$

$x^2+x-6 = x^2-3x-4+2$

$x-6 = -3x-2$

$4x = 4$

$x = 1$

No! Not in Case I

Case III: $-1 \le x \le 2$

$-x^2-x+6 = -x^2+3x+4+2$

$-x+6 = 3x+6$

$0 = 4x$

$\boxed{x=0}$ yes!

Case V: $x \ge 4$

$x^2+x-6 = x^2-3x-4+2$

$x-6 = -3x-2$

$4x = 4$

$x = 1$

No! Not in Case V

Case II: $-3 \le x \le -1$

$-x^2-x+6 = x^2-3x-4+2$

$0 = 2x^2-2x-8$

$0 = x^2-x-4$

$x = \dfrac{1 \pm \sqrt{1-4(1)(-4)}}{2(1)}$

$x = \dfrac{1 \pm \sqrt{17}}{2}$

$x = \dfrac{1+\sqrt{17}}{2}$ No... not in case II

$\boxed{x = \dfrac{1-\sqrt{17}}{2}}$ yes... in case II

Case IV: $2 \le x \le 4$

$x^2+x-6 = -x^2+3x+4+2$

$2x^2-2x-12 = 0$

$x^2-x-6 = 0$

$(x-3)(x+2) = 0$

$\boxed{x=3}$ yes!

or

$x = -2$ No! Not in Case IV

Overall: $x = 0$, $x = 3$, $x = \dfrac{1-\sqrt{17}}{2}$

4.) See the attached function $y = f(x)$ in **Figure 1**. Specifically state what transformations need to be performed to $y = f(x)$ in order to graph the function $y = -f\left(1 - \frac{x}{3}\right) + 2$. Then, sketch the graph of the new function <u>on the same grid</u>. **[6 Points]**

$y = -f\left[-\frac{1}{3}(x-3)\right] + 2$

Rx, Ry, HSTR 3, → 3, ↑ 2

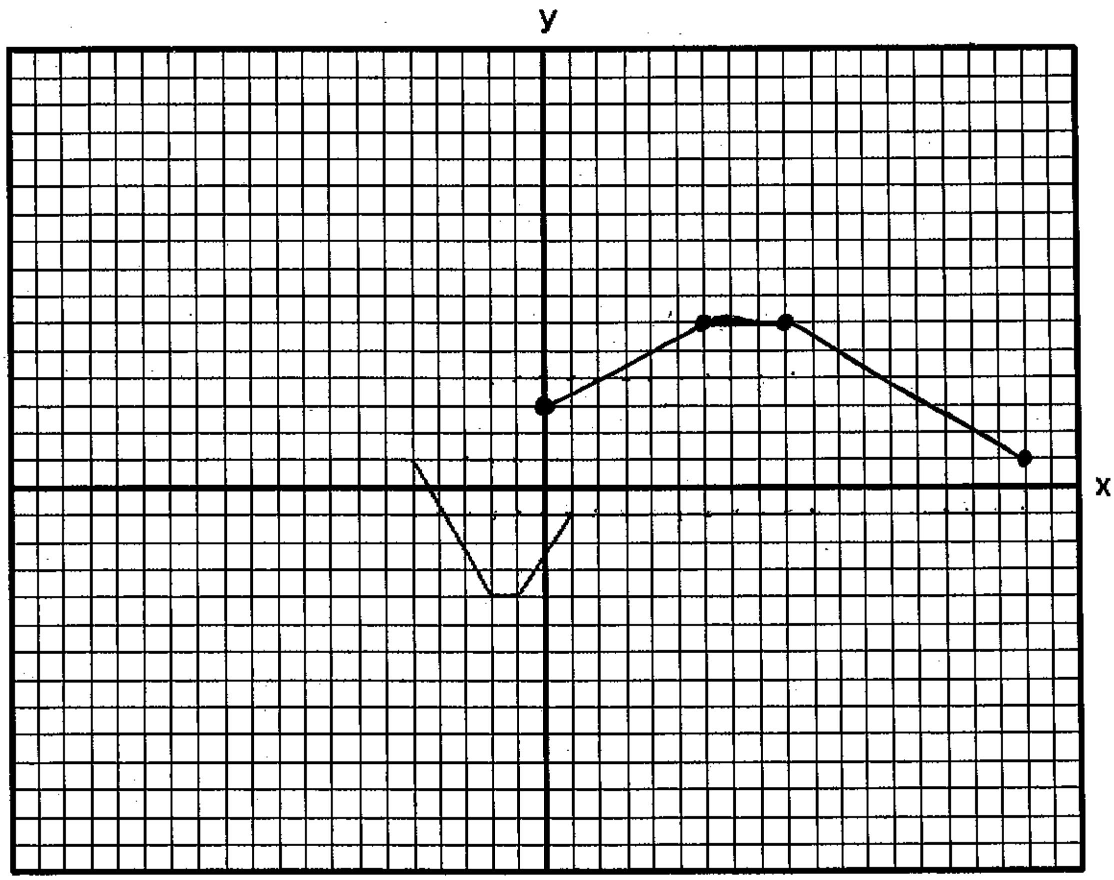


Figure 1

5.) For **Figure 2**, do what is requested <u>where the questions are stated</u>. (Please do not do this problem on separate paper.) **[9 Points]**

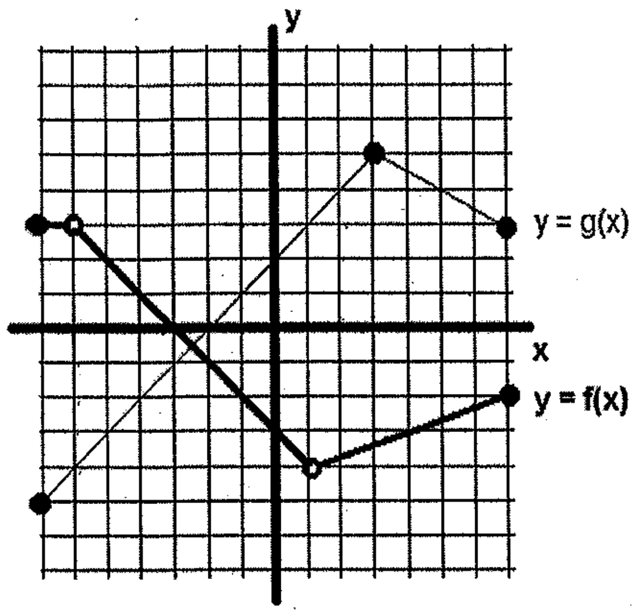


Figure 2

Domain of $f(x)$: $[-7,-6)\cup(-6,1)\cup(1,7]$

Range of $f(x)$: $(-4,3]$

Domain of $g(x)$: $[-7,7]$

Range of $g(x)$: $[-5,5]$

Show the steps used to arrive at your answer.

$(f \circ g)(2) =$ $f(4) = \boxed{-3}$

$(g \circ f)(7) =$ $g(-2) = \boxed{0}$

$(f \circ g)(-6) =$ $f(-4) = \boxed{1}$

$(g \circ f)(-1) =$ $g(-2) = \boxed{0}$

$f(x) = 1 \quad x = ?$

$\boxed{-4}$

6.) Given that the point $\left(-\frac{3}{7}, \frac{2}{3}\right)$ is on the graph of $y=f(x)$, what point must fall on the graph of the function $y=-\frac{9}{4}f\left(\frac{2x-5}{8x+1}+1\right)+2$? **[6 Points]**

x (input)

$f\left(-\frac{3}{7}\right) = \frac{2}{3}$

y (output)

$\frac{2x-5}{8x+1}+1 = -\frac{3}{7}$

$\frac{2x-5}{8x+1} = \frac{-10}{7}$

$7(2x-5) = -10(8x+1)$

$14x-35 = -80x-10$

$94x = 25$

$x = \frac{25}{94}$

$y = -\frac{9}{4}\left(\frac{2}{3}\right)+2$

$y = -\frac{3}{2}+2$

$y = \frac{1}{2}$

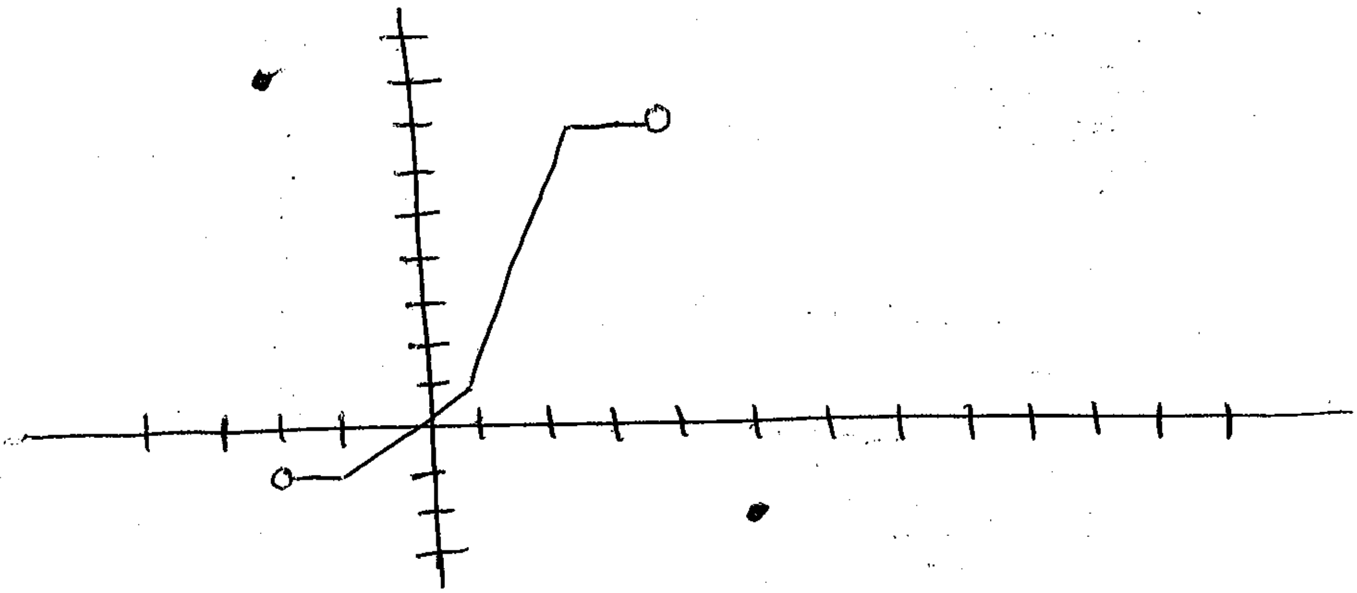
$\boxed{\text{Point: } \left(\frac{25}{94}, \frac{1}{2}\right)}$

7.) Draw any function with the following properties: **[5 Points]**

Domain: [-2, 4) U {5}

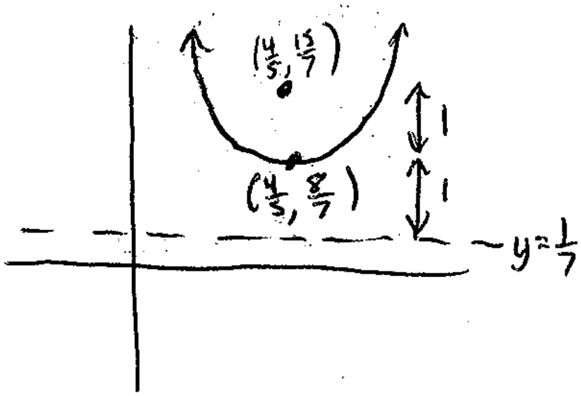
Range: {-2, 8} U [-1, 7]

one possibility:



⑧ See next page

9.) Find the equation of the parabola having focal point $\left(\frac{4}{5}, \frac{15}{7}\right)$ and directrix $y=\frac{1}{7}$. **[4 Points]**



$c=1$

$y=\frac{x^2}{4(1)}$ Right $\frac{4}{5}$, Up $\frac{8}{7}$

$$\boxed{y=\frac{(x-\frac{4}{5})^2}{4}+\frac{8}{7}}$$

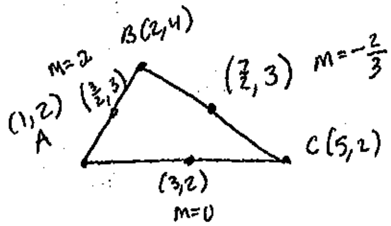
8.) Given ΔABC with $A(1, 2)$, $B(2, 4)$, $C(5, 2)$, find the following: **[9 Points]**

A.) Coordinates of Circumcenter (where perpendicular bisectors intersect)

B.) Coordinates of Orthocenter (where altitudes intersect)

C.) Coordinates of Centroid (where medians intersect)

Also, show that all of its perpendicular bisectors intersect at the circumcenter, all of its altitudes intersect at the orthocenter, and all of its medians intersect at the centroid.



1.) Circumcenter: $\left(3, \frac{9}{4}\right)$

A.) $\perp \overline{AB}$: $m=-\frac{1}{2}$ $\left(\frac{3}{2}, 3\right)$

$y-3=-\frac{1}{2}\left(x-\frac{3}{2}\right)$

$y-3=-\frac{1}{2}x+\frac{3}{4}$

$\boxed{y=-\frac{1}{2}x+\frac{15}{4}}$

B.) $\perp \overline{AC}$: $\boxed{x=3}$

$m=$ undef.

$(3,2)$

C.) $\perp \overline{BC}$: $m=\frac{3}{2}$ $\left(\frac{7}{2}, 3\right)$

$y-y_1=m(x-x_1)$

$y-3=\frac{3}{2}\left(x-\frac{7}{2}\right)$

$y-3=\frac{3}{2}x-\frac{21}{4}$

$\boxed{y=\frac{3}{2}x-\frac{9}{4}}$

$\left(3, \frac{9}{4}\right)$

$y=-\frac{1}{2}(3)+\frac{15}{4}$

$y=-\frac{3}{2}+\frac{15}{4}$

$y=\frac{9}{4}$

$\begin{cases} y=-\frac{1}{2}x+\frac{15}{4} \\ x=3 \\ y=\frac{3}{2}x-\frac{9}{4} \end{cases}$

$y=\frac{3}{2}(3)-\frac{9}{4}$

$y=\frac{9}{2}-\frac{9}{4}$

$y=\frac{9}{4}$

2.) Orthocenter $\left(2, \frac{7}{2}\right)$

A.) $\overline{AB}$: $m=-\frac{1}{2}$ $(5,2)$

$y-y_1=m(x-x_1)$

$y-2=-\frac{1}{2}(x-5)$

$y-2=-\frac{1}{2}x+\frac{5}{2}$

$\boxed{y=-\frac{1}{2}x+\frac{9}{2}}$

B.) $\overline{AC}$: $m=$ undefined $(2,4)$

$\boxed{x=2}$

C.) $\overline{BC}$: $m=\frac{3}{2}$ $(1,2)$

$y-y_1=m(x-x_1)$

$y-2=\frac{3}{2}(x-1)$

$y-2=\frac{3}{2}x-\frac{3}{2}$

$y=\frac{3}{2}x+\frac{1}{2}$

$\begin{cases} y=-\frac{1}{2}x+\frac{9}{2} \\ x=2 \\ y=\frac{3}{2}x+\frac{1}{2} \end{cases}$

$y=\frac{3}{2}(2)+\frac{1}{2}$

$y=\frac{7}{2}$

$y=-\frac{1}{2}(2)+\frac{9}{2}$

$y=-1+\frac{9}{2}$

$y=\frac{7}{2}$

Centroid

$\left(\frac{8}{3}, \frac{8}{3}\right)$

3.) Centroid

A.) $\overline{AB}$: $\left(\frac{3}{2}, 3\right), (5,2)$ $\quad m=\frac{2-3}{5-\frac{3}{2}}=\frac{-1}{\left(\frac{7}{2}\right)}=-\frac{2}{7}$

$m=-\frac{2}{7}; (5,2) \rightarrow$ $y-y_1=m(x-x_1)$

$y-2=-\frac{2}{7}(x-5)$

$y-2=-\frac{2}{7}x+\frac{10}{7}$

$y=-\frac{2}{7}x+\frac{24}{7}$

B.) $\overline{AC}$: $(3,2); (2,4) \longrightarrow m=\frac{4-2}{2-3}=\frac{2}{-1}=-2$

$m=-2; (3,2) \longrightarrow$ $y-y_1=m(x-x_1)$

$y-4=-2(x-3)$

$y-2=-2x+6$

$y=-2x+8$

C.) $\left(\frac{7}{2}, 3\right), (1,2) \rightarrow m=\frac{2-3}{1-\frac{7}{2}}=\frac{-1}{-\frac{5}{2}}=\frac{2}{5}$

$m=\frac{2}{5}, (1,2) \longrightarrow$ $y-y_1=m(x-x_1)$

$y-2=\frac{2}{5}(x-1)$

$y-2=\frac{2}{5}x-\frac{2}{5}$

$y=\frac{2}{5}x+\frac{8}{5}$

$\begin{cases} y=-\frac{2}{7}x+\frac{24}{7} \\ y=-2x+8 \\ y=\frac{2}{5}x+\frac{8}{5} \end{cases}$

$\rightarrow -2x+8=\frac{2}{5}x+\frac{8}{5} \rightarrow -10x+40=2x+8$

$-12x=-32$

$x=\frac{8}{3}$

$y=-2\left(\frac{8}{3}\right)+8$

$y=\frac{8}{3}$

$x=\frac{8}{3}$

10.) Find the center point and radius of the circle **[4 Points]**

$$16x^2+16y^2+64x+65=8y+100 \ .$$

$$16x^2+64x \quad +16y^2-8y \quad +65 = 100$$

$$16(x^2+4x+\overset{+64}{4})-64 \quad +16\left(y^2-\tfrac{1}{2}y+\tfrac{1}{16}\right)-1 \quad +65 = 100$$

$$16(x+2)^2-64 \quad +16\left(y-\tfrac{1}{4}\right)^2 -1 \quad +65=100$$

$$16(x+2)^2 + 16\left(y-\tfrac{1}{4}\right)^2 = 100$$

$$(x+2)^2+\left(y-\tfrac{1}{4}\right)^2 = \frac{100}{16}$$

$$(x+2)^2+\left(y-\tfrac{1}{4}\right)^2 = \frac{25}{4}$$

$$\boxed{\begin{array}{l} C=\left(-2,\tfrac{1}{4}\right) \\ r=\tfrac{5}{2} \end{array}}$$

11.) Graph the following, solving for (and labeling) all of its key features **[10 Points]**

$$36x^2+288x+25y^2-50y=299 \ .$$

$$36(x^2+8x+\overset{576}{16})-576 \quad +25(y^2-2y+\overset{25}{1})-25 = 299$$

$$36(x+4)^2-576 \quad +25(y-1)^2-25 = 299$$

$$36(x+4)^2+25(y-1)^2=900$$

Ellipse (Vertical)

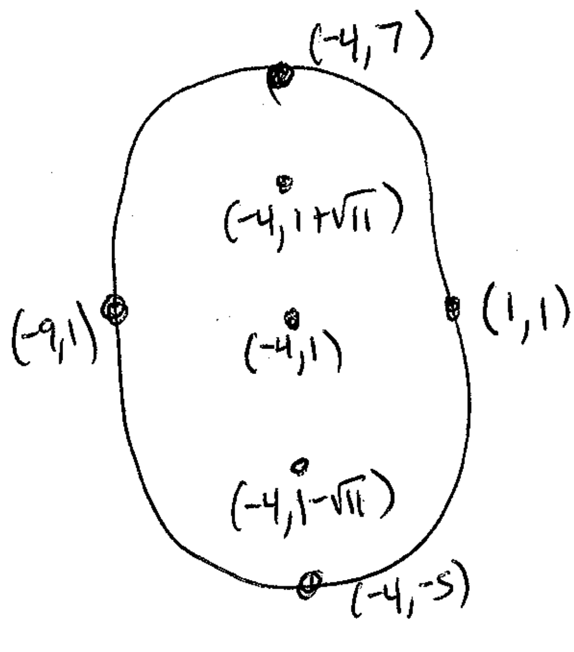
Ellipse $\boxed{\dfrac{(x+4)^2}{25}+\dfrac{(y-1)^2}{36}=1}$

$C(-4,1)$

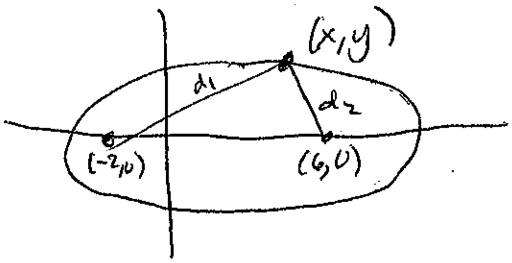
$\leftrightarrow 5 \quad \updownarrow 6$

$c=\sqrt{36-25}$

$c=\sqrt{11}$



12.) Derive the equation of the ellipse with focal points (-2, 0) and (6, 0) that has a constant sum of 10. You must do the full derivation (with distance formula) for full credit! **[5 Points]**



$$d_1 + d_2 = 10$$

$$\sqrt{(x+2)^2 + y^2} + \sqrt{(x-6)^2 + y^2} = 10$$

$$\left(\sqrt{(x+2)^2 + y^2}\right)^2 = \left(10 - \sqrt{(x-6)^2 + y^2}\right)^2$$

$$(x+2)^2 + y^2 = 100 - 20\sqrt{(x-6)^2 + y^2} + (x-6)^2 + y^2$$

$$x^2 + 4x + 4 + y^2 = 100 - 20\sqrt{(x-6)^2 + y^2} + x^2 - 12x + 36 + y^2$$

$$4x + 4 = -12x + 136 - 20\sqrt{(x-6)^2 + y^2}$$

$$16x - 132 = -20\sqrt{(x-6)^2 + y^2}$$

$$(4x - 33)^2 = \left(-5\sqrt{(x-6)^2 + y^2}\right)^2$$

$$16x^2 - 264x + 1089 = 25\left[(x-6)^2 + y^2\right]$$

$$16x^2 - 264x + 1089 = 25\left[x^2 - 12x + 36 + y^2\right]$$

$$16x^2 - 264x + 1089 = 25x^2 - 300x + 900 + 25y^2$$

$$189 = 9x^2 - 36x + 25y^2$$

$$189 = 9(x^2 - 4x + \overset{+36}{4}) - 36 + 25y^2$$

$$189 = 9(x-2)^2 - 36 + 25y^2$$

$$\frac{225}{225} = \frac{9(x-2)^2}{225} + \frac{25y^2}{225}$$

$$1 = \frac{(x-2)^2}{25} + \frac{y^2}{9}$$

$$\boxed{\frac{(x-2)^2}{25} + \frac{y^2}{9} = 1}$$

13.) Find the system of inequalities needed to form the region in **Figure 3**. **[4 Points]**

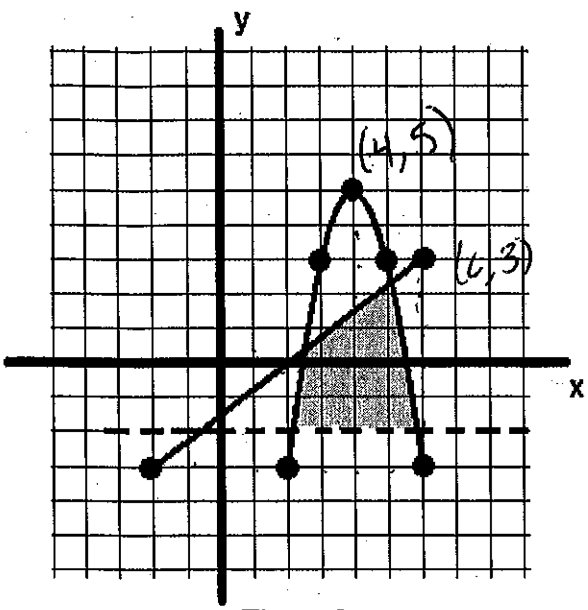


Figure 3

Parabola: $y = -2(x-4)^2 + 5$

Below

$y \le -2(x-4)^2 + 5$

Dotted Line: $y = -2$

Above

$y > -2$

Solid Line: $m = \frac{6}{8} = \frac{3}{4}$

$m = \frac{3}{4};\ (6,3)$

$y - 3 = \frac{3}{4}(x-6)$

$y - 3 = \frac{3}{4}x - \frac{9}{2}$

$y = \frac{3}{4}x - \frac{3}{2}$

Below

$y \le \frac{3}{4}x - \frac{3}{2}$

Answer:

$$\boxed{\begin{cases} y \le -2(x-4)^2 + 5 \\ y > -2 \\ y \le \frac{3}{4}x - \frac{3}{2} \end{cases}}$$

14.) In **Figure 4**, SQRE is a square of side length a and ΔSMQ is equilateral. Find the area of ΔQCR in <u>exact</u> form (simplified square roots and fractions, where applicable). **[2 Points]**

one approach:

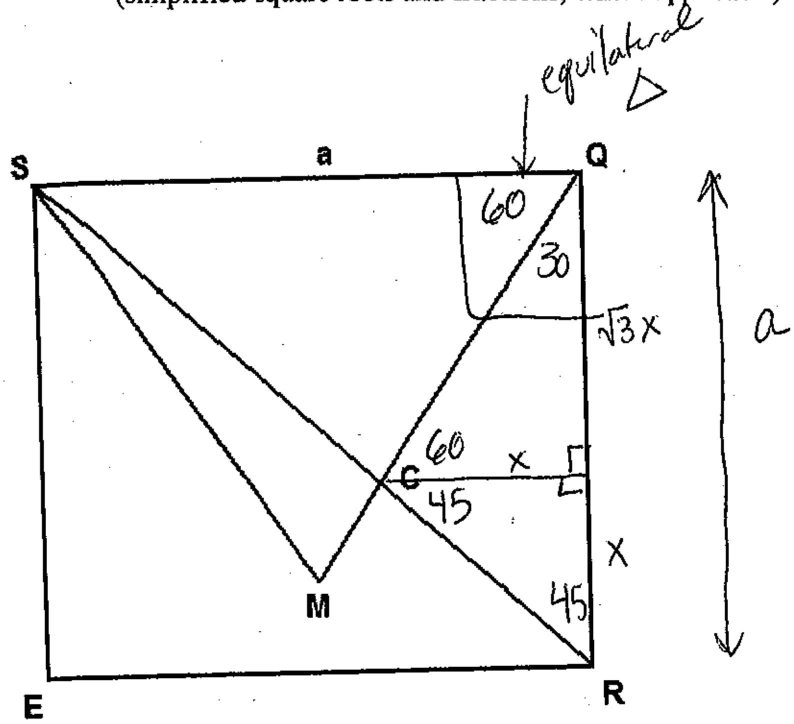


Figure 4

• $\overline{SR}$ cuts $\angle QRE$ in half (Diagonal of Square)

• Let x = the height of ΔQCR.

Then

$$x + \sqrt{3}x = a$$

$$x(1+\sqrt{3}) = a$$

$$x = \frac{a}{1+\sqrt{3}}\left(\frac{1-\sqrt{3}}{1-\sqrt{3}}\right)$$

$$x = \frac{a(1-\sqrt{3})}{1-3}$$

$$x = \frac{a(1-\sqrt{3})}{-2}$$

$$x = \frac{a(\sqrt{3}-1)}{2}$$

Area of ΔQCR

$$= \frac{1}{2}\cdot a \cdot \left(\frac{a(\sqrt{3}-1)}{2}\right)$$

$$= \boxed{\frac{a^2(\sqrt{3}-1)}{4}}$$